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RESEARCH ARTICLE

EW Radiative Corrections to $e^+e^- \to \tilde{\chi}_1^0\, \tilde{\chi}_1^0 h^0$ for Different Scenarios

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ABSTRACT

In this work, the radiative corrections to the production of a light neutral Higgs boson (h^0) with a pair of lightest neutralinos ($\tilde{\chi}_1^0$) in e^+e^- collisions within MSSM are presented through two different SUSY scenarios –Higgsino and Gaugino scenarios-, including the on-shell renormalization scheme in the loop calculations. We have studied the QED corrections as well as the weak corrections, where the contribution from both corrections is significant and needs to be taken into account in the future linear colliders experiments. The result includes the numerical calculations for $e^+e^- \to \tilde{\chi}_1^0\, \tilde{\chi}_1^0 h^0$.

Key words: Higgs boson, Neutralino, EW Radiative Corrections, QED, MSSM.

INTRODUCTION

On July 2012, both CMS and ATLAS collaborations at the LHC, announced independently, that they both discovered a previously unknown boson of mass between 125 and 127 GeV (Cho, 2012; The ATLAS Collaboration, 2012), and it is confirmed by experiments to be the Higgs boson, on March 2013. Since the discovery of the Higgs boson, the experimentalists as well as theorists expected to discover the rest of the Minimal Super symmetric Standard Model (MSSM) particles and support the validity of Super symmetry (SUSY) theory. SUSY is a type of space-time symmetry between bosons and fermions. In realistic models, SUSY is broken at the weak scale implying that all Standard Model (SM) particles must have super partners with masses in the range $\sim 100-1000$ GeV and up to 2 TeV that will be accessible to colliders (Bagnaschi *et al*., 2017). The discovery of Higgs boson has impact on the search for particles such as neutralino (Beskidt *et al*., 2014), where the couplings of the Higgs bosons to the SUSY scalar fermions $\tilde{f}$ and to the charginos $\tilde{\chi}^{\pm}$ and neutralinos $\tilde{\chi}^0$ depend on the soft–SUSY breaking parameters and therefore carry information on the fundamental SUSY theory. Searches for direct detection of dark matter have focused primarily on the weakly interacting massive particles (WIMPs) and more precisely on the lightest super symmetric particles (LSPs). These are hypothetical particles such as neutralinos that are least massive members of the hypothesized family of supersymmetric partner particles. In addition to consider the neutralinos as one of the best candidates for the dark matter, the studying of neutralinos masses presents lots of information on the SUSY-breaking structure. In MSSM (Nilles, 1984), one has four neutralinos $\tilde{\chi}_1^0$-$\tilde{\chi}_4^0$, which are the fermion mass eigenstates of the supersymmetric partners of the photon, the Z^0 boson, and the neutral Higgs bosons $H_{1,2}^0$. Their mass matrix depends on the parameters M_1, M_2, μ, and $\tan\beta$, where $M_1$and M_2 are the SU(2) and U(1) gauge mass parameter, $\tan\beta = v_1/v_2$with $v_{1,2}$ the vacuum expectation values of the two neutral Higgs doublet fields. If supersymmetry is realized in nature, neutralinos should be found in the present high energy experiments at Tevatron, LHC (The ATLAS Collaboration, 2014) and future e^+e^- colliders. Especially at a linear e^+e^- collider, it will be possible to perform measurements with high precision (TESLA Technical Design Report, 2001-011; Adolphsen *et al*., 2000). To obtain high matching between experiments results and theoretical prediction, it is inevitable to include higher-order terms in the calculation of the measured quantities, since the previous studies have showed that the Born-level evaluations can be affected significantly by one-loop radiative corrections.

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In this paper, we use on-shell renormalization scheme in the loop calculations of the Higgs and neutralino sectors of the CP-conserving MSSM. The calculation was performed using the FeynArts and FormCalc computer packages. All the renormalization constants, required to determine the various counterterms for the Higgs, neutralino and other sectors, being implemented in the MSSM version of Feyn Arts (Hahn and Schappacher, 2002) for completion at the one-loop level. The resulting amplitudes were algebraically simplified using Form Calc and then converted to a FORTRAN program. The Loop Tools package was used to evaluate the one-loop scalar and tensor integrals (Hahn, 2000).

Radiative Corrections

The neutralino pair production in association with MSSM neutral Higgs boson can be presented as following:

$$e^+(p_1) + e^-(p_2) \to \tilde{\chi}_1^0(p_3) + \tilde{\chi}_1^0(p_4) + h^0(p_5),$$

where the momenta of the particles are given in brackets and

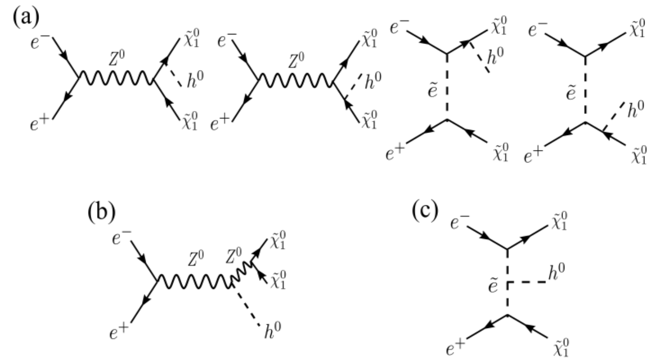

Figure 1. The lowest order (LO) Feynman diagrams for the $e^+e^- \to \tilde{\chi}_1^0\,\tilde{\chi}_1^0 h^0$

obey the on-shell conditions $p_1^2 = p_2^2 = 0$, $p_3^2 = p_4^2 = m_{\tilde{\chi}_i^0}^2$, $p_5^2 = m_h^2$. The center-of-mass energy squared $s = (p_1 + p_2)^2$.

Feynman diagrams in Fig. 1 represent the 6 most contributing topologies involved in this process at tree level:

- 3 with the s-channel Z^0 exchange,
- 3 with the t-channel left– and right–handed selectron $\tilde{e}_{L,R}$ exchange.

The diagrams where the h^0 boson is emitted from the electron and positron lines give negligible contributions.

From the interaction Lagrangian corresponding to Fig. 1 (Gunion and Haber, 1986; Haber and Kane, 1985):

$$\mathcal{L}_{Z^0\bar{e}e} = -\frac{g}{\cos\theta_W} Z_\mu^0 \bar{e}\,\gamma^\mu [C_L P_L + C_R P_R] e,$$

$$\mathcal{L}_{Z^0\tilde{\chi}_i^0\tilde{\chi}_j^0} = \frac{g}{2\cos\theta_W} Z_\mu^0 \bar{\tilde{\chi}}_i^0 \gamma^\mu \left[O_{ij}^{''L} P_L + O_{ij}^{''R} P_R\right] \tilde{\chi}_j^0,$$

$$\mathcal{L}_{e\tilde{e}\tilde{\chi}_j^0} = g f_i^L \bar{e} P_R \tilde{\chi}_i^0 \tilde{e}_L + g f_i^R \bar{e} P_L \tilde{\chi}_i^0 \tilde{e}_R + h.c.,$$

$$\mathcal{L}_{h^0\tilde{\chi}_i^0\tilde{\chi}_j^0} = -\frac{g}{2} h^0 \cos\alpha\, \bar{\tilde{\chi}}_i^0 \left(Q_{ij}^{''*} P_L + Q_{ij}^{''} P_R\right) \tilde{\chi}_j^0 + \frac{g}{2} h^0 \sin\alpha\, \bar{\tilde{\chi}}_i^0 \left(S_{ij}^{''*} P_L + S_{ij}^{''} P_R\right) \tilde{\chi}_j^0,$$

$$\mathcal{L}_{h^0 ZZ} = \frac{g m_Z}{2\cos\theta_W} Z_\mu Z^\mu h^0 \cos(\beta - \alpha),$$

$$\mathcal{L}_{h^0\tilde{e}\tilde{e}} = \frac{g m_Z}{\cos\theta_W} h^0 \left[(I^{3L} - e_{\tilde{e}} \sin^2\theta_W) \tilde{e}_L^* \tilde{e}_L + e_{\tilde{e}} \sin^2\theta_W \tilde{e}_R^* \tilde{e}_R\right],$$

one obtains the following couplings:

$$C_{L,R} = I^{3L,R} + \sin^2\theta_W, \quad I^{3L} = -\frac{1}{2}, \quad I^{3R} = 0$$

$$O_{ij}^{''L} = -O_{ij}^{''R*} = -\frac{1}{2} N_{i3} N_{j3}^* + \frac{1}{2} N_{i4} N_{j4}^*,$$

$$f_i^L = -\frac{\sqrt{2}}{2} (\tan\theta_W N_{i1} + N_{i2}), \quad f_i^R = \sqrt{2} \tan\theta_W N_{i1}^*,$$

$$g Q_{ij}^{''} = \frac{1}{2} \left[N_{i3} \left(g N_{j2} - g' N_{j1}\right) + \sqrt{2}\, h^* N_{i4} N_{j5} + (i \leftrightarrow j)\right],$$

$$g S_{ij}^{''} = \frac{1}{2} \left[N_{i4} \left(g N_{j2} - g' N_{j1}\right) - \sqrt{2}\, h^* N_{i3} N_{j5} + (i \leftrightarrow j)\right],$$

$$R_{ij}^{''} = \frac{1}{2 m_W} \left[M^* N_{i2} N_{j2} + M'^* N_{i1} N_{j1} - M^* \left(N_{i3} N_{j4} + N_{i4} N_{j3}\right)\right],$$

For high precise results, radiative corrections should be included in the calculations which involve virtual one-loop correction and real photon emission such that:

$$\sigma = \sigma^{virt} + \sigma^{real} \tag{2}$$

$$= \int \sum_{spin} |\mathcal{M}_{virt}(e^+e^- \to \tilde{\chi}_1^0 \tilde{\chi}_1^0 h^0)|^2 d\Phi_3 + \int \sum_{spin} |\mathcal{M}_{real}(e^+e^- \to \tilde{\chi}_1^0 \tilde{\chi}_1^0 h^0 \gamma)|^2 d\Phi_4$$

Virtual Corrections

The set of Feynman diagrams in Fig. 1, has to be dressed by the corresponding loop contributions containing the full particle spectrum of the MSSM. One-loop Feynman diagrams can be classified as the following generic structure: The virtual vertex corrections Fig. 2, the box graph contributions to the propagators Fig. 3, and the self-energy contributions Fig. 4. The complete supersymmetric spectrum is used for the virtual particles inside loops. The evaluation of one-loop diagrams usually leads to two types of divergences:

- UV divergences, which are associated with singularities occurring at large loop momenta,
- IR divergences, which are generated, if one of the propagators in the loop vanishes.

To isolate the UV divergences, the regularization by dimensional reduction scheme (DR) is used to preserve SUSY. In this scheme only the momenta are treated as *D*-dimensional, while the fields and the Dirac algebra are kept 4-dimensional. To get rid of the UV divergences and absorb them, they should be renormalized by introducing a suitable set of counterterms for the renormalization of the coupling constants and the renormalization of the external wave functions. In this paper on-shell renormalization scheme is used in which all particle masses are defined as pole masses, such that the cross sections are directly related to the physical masses of the external particles and the other particles entering the loops (Choi *et al.*, 2000; Fritzsche and Hollik, 2002). The complete cross section at the one-loop level can be written as follows:

$$\sigma^{1-loop} = \sigma^0 + \sigma^{virt} \tag{3}$$

The virtual electroweak radiative correction to the cross section is given by

$$\sigma^{virt} = \sigma^0 \Delta_{virt} = \frac{(2\pi)^4}{2|\vec{p}_1|\sqrt{s}} \int d\Phi_3 \sum_{spin} Re\left(\mathcal{M}_0^\dagger \mathcal{M}_{virt}\right), \tag{4}$$

where Δ_{virt} is the relative virtual correction and $\mathcal{M}_{virt}$ is the renormalized amplitude involving all the one-loop electroweak Feynman diagrams and corresponding counterterms. The contributions of virtual photon exchange in loops leads to soft IR divergences as well as the real photon emission (Fritzsche and Hollik, 2004), but their sum is IR finite.

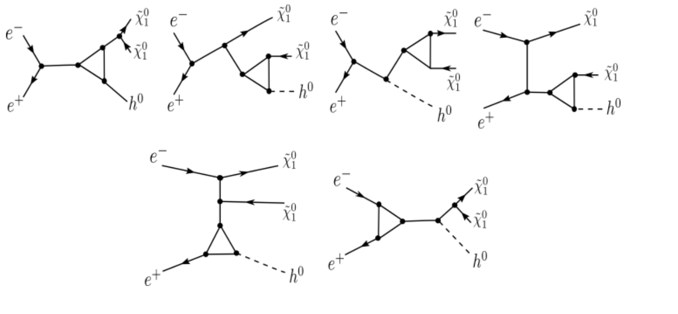

Figure 2. Vertex Corrections

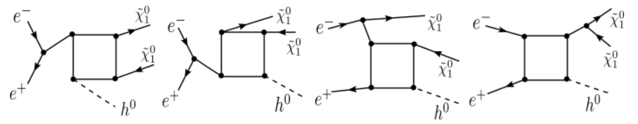

Figure 3. Box Corrections

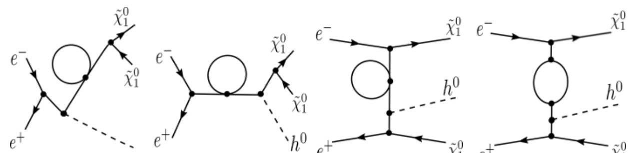

Figure 4. Self Corrections

From previous discussion the corrected cross section can be expressed as following:

$$\sigma^{corr}(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_1^0 h^0)$$

$$= \sigma^{ren}(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_1^0 h^0) + \sigma(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_1^0 h^0\gamma)$$

Renormalization of Neutralino Sector

The tree level neutralino mass terms are given by:

$$\mathcal{L}_{\rm n} = -\tfrac{1}{2}\left[\psi^{0^{\intercal}} Y\, \psi^0 + \bar{\psi}^{0^{\intercal}} Y^{\dagger} \bar{\psi}^0\right] + h.c.,$$

Where

$$\psi^0 = \left(\tilde{B}^0, \widetilde{W}^3, \tilde{h}_1^0, \tilde{h}_2^0\right)^T$$

Lagrangian involves the μ parameter, the soft-breaking gaugino-mass parameters M_1 and M_2, and the Higgs vacua v_i, which are related to $\tan\beta = v_2/v_1$ and to the W mass $M_W = gv/2$ with $(v_1^2 + v_2^2)^{1/2}$ (Fritzsche and Hollik, 2002; Dabelstein, 1995). After the electroweak symmetry is broken, the neutralino mass matrix in the bino–wino–higgsino basis can be written as:

$$Y = \begin{pmatrix} M_1 & 0 & -M_Z\, s_W \cos\beta & M_Z\, s_W \sin\beta \\ 0 & M_2 & M_Z\, c_W \cos\beta & -M_Z\, c_W \sin\beta \\ -M_Z\, s_W \cos\beta & M_Z\, c_W \cos\beta & 0 & -\mu \\ M_Z\, s_W\, sin\,\beta & -M_Z\, c_W\, sin\,\beta & -\mu & 0 \end{pmatrix},$$

which can be diagonalized with the help of a unitary 4×4 matrix N, yielding the neutralino mass eigenstates $\tilde{\chi}_i^0 (i = 1,\cdots,4)$.

Renormalization constants are introduced for the neutralino mass matrix Y and for the neutralino fields ψ^0 by the transformation:

$$Y \to Y + \delta Y,$$
$$\psi^0 \to \left(1 + \tfrac{1}{2}\delta Z_{\tilde{\chi}^0}\right)\psi^0, \tag{5}$$

where the matrix-valued renormalization constant $\delta Z_{\tilde{\chi}^0}$ is a general complex 4×4 matrix of one-loop order. The physical (on-shell) masses are defined as poles of the real parts of the one-loop corrected propagators. The physical neutralino masses are then given by,

$$m_{\tilde{\chi}_i^0}^{os} = m_{\tilde{\chi}_i^0} + (N^* \delta Y N^{-1})_{ii} - \delta m_{\tilde{\chi}_i^0}, \tag{6}$$

where $m_{\tilde{\chi}_i^0}$ is the finite tree level mass, and $\delta m_{\tilde{\chi}_i^0}$ is the loop correction to the neutralino mass. The pole mass $m_{\tilde{\chi}_i^0}^{os}$ is considered as an input by specification of the parameters μ, M_1, M_2, which are related to the input masses in the same way as in LO. In this way, the tree-level masses $m_{\tilde{\chi}_i^0}$ as well as the counterterm matrix δY, are fixed.

The matrix δY consists of the counterterms for the following parameters in the mass matrix Y: M_1, M_2, μ, $\tan\beta$, the Z boson mass M_Z, W boson mass, which is involved in θ_W, and the electroweak mixing angle $s_W = \sin\theta_W$, $c_W = \cos\theta_W$, such that:

$$\delta Y = \begin{pmatrix} \delta M_1 & 0 & \delta Y_{13} & \delta Y_{14} \\ 0 & \delta M_2 & \delta Y_{23} & \delta Y_{24} \\ -\delta Y_{31} & \delta Y_{32} & 0 & -\delta\mu \\ \delta Y_{41} & -\delta Y_{42} & -\delta\mu & 0 \end{pmatrix}. \tag{7}$$

δM_W^2, δM_Z^2 and $\delta\theta_W$ are the same as in SM. We renormalize them according to the on–shell prescription of electroweak renormalization, where M_W and M_Z are physical (pole) masses, and $\cos\theta_W = M_W / M_Z$. This gives (Farzinnia and He, 2013):

$$\delta M_W^2 = \widetilde{\Re\mathrm{e}}\textstyle\sum_{WW}(M_W^2);$$

$$\delta M_Z^2 = \widetilde{\Re\mathrm{e}}\textstyle\sum_{ZZ}(M_Z^2);$$

$$\delta\cos\theta_W = \frac{M_W}{M_Z}\left(\frac{\delta M_W}{M_W} - \frac{\delta M_Z}{M_Z}\right). \tag{8}$$

$\sum_{WW}$ and $\sum_{ZZ}$ are the transverse components of the diagonal W and Z two–point functions in momentum space, respectively. Those three counter terms have, besides the contributions from the SM, new contributions from the MSSM involving loops of super particles and additional Higgs bosons. $\delta\tan\beta$ is fixed in Higgs sector as following:

$$\delta\tan\beta = \frac{1}{2M_Z\cos^2\beta}\Im\mathfrak{m}\left(\textstyle\sum_{AZ}(m_A^2)\right), \tag{9}$$

this implies that the two-point function connecting the CP-odd Higgs boson A to Z boson vanishes when A is on-shell.

δM_1, δM_2 and $\delta\mu$ are fixed in neutralinos sector. By using the three neutralino masses as inputs, the counterterms δM_1, δM_2 and $\delta\mu$ are all determined from Eq. (6).

Renormalization of Higgs Sector

The MSSM requires two Higgs doublets H_1 and H_2 with opposite hypercharge $Y_1 = -Y_2 = -1$. The quadratic part of the Higgs potential in the MSSM is given by:

$$V = m_1^2 H_1\bar{H}_1 + m_2^2 H_2\bar{H}_2 + m_{12}^2(\epsilon_{ab}H_1^a H_2^b + h.c.)$$
$$+\frac{1}{8}(g_1^2+g_2^2)(H_1\bar{H}_1 - H_2\bar{H}_2)^2 - \frac{g_2^2}{2}|H_1\bar{H}_2|^2, \qquad (10)$$

where m_{12}^2 is defined to be negative and $\epsilon_{12} = -\epsilon_{21} = -1$, with soft breaking parameters m_1^2, m_2^2, m_{12}^2 and g_1, g_2 are, respectively, $\mathrm{SU}(2)$ and $\mathrm{U}(1)$ gauge couplings. Decomposing each Higgs doublet field $H_{1,2}$ in terms of its components (Fritzsche and Hollik, 2004; Dabelstein, 1995), we get

$$H_1 = \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} (v_1 + \phi_1^0 - i\chi_1^0)/\sqrt{2} \\ -\phi_1^- \end{pmatrix},$$
$$H_2 = \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ (v_2 + \phi_2^0 + i\chi_2^0)/\sqrt{2} \end{pmatrix}, \qquad (11)$$

with vacuum expectation values v_1, v_2.

The Higgs potential (10) is diagonalized by the rotations

$$\begin{pmatrix} H^0 \\ h^0 \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}\begin{pmatrix} \phi_1^0 \\ \phi_2^0 \end{pmatrix}$$
$$\begin{pmatrix} G^0 \\ A^0 \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix}\begin{pmatrix} \chi_1^0 \\ \chi_2^0 \end{pmatrix}$$
$$\begin{pmatrix} G^+ \\ H^+ \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix}\begin{pmatrix} \phi_1^+ \\ \phi_2^+ \end{pmatrix}. \qquad (12)$$

$G^0, G^\pm$ describe the unphysical Goldstone modes. The spectrum of physical states consists of: a light neutral CP-even state (h^0), a heavy neutral CP-even state (H^0), a neutral CP-odd state (A^0), and a pair of charged states ($H^\pm$). The masses of the gauge bosons and the electromagnetic charge are determined by:

$$M_Z^2 = \frac{1}{4}(g_1^2+g_2^2)(v_1^2+v_2^2)\,,$$
$$M_W^2 = \frac{1}{4}g_2^2(v_1^2+v_2^2)\,,$$
$$e^2 = \frac{g_1^2 g_2^2}{g_1^2+g_2^2}\,. \qquad (13)$$

Thus, the potential (10) contains two independent free parameters, which can conveniently be chosen as

$$\tan\beta = \frac{v_2}{v_1}\,, \quad M_A^2 = -m_{12}^2(\tan\beta + \cot\beta), \qquad (14)$$

where M_A is the mass of the A^0 boson.

Expressed in terms of Eq. (14), the masses of the other physical states are written as:

$$m_{H^0,h^0}^2 = \frac{1}{2}\left(M_A^2 + M_Z^2 \pm \sqrt{(M_A^2+M_Z^2)^2 - 4M_A^2 M_Z^2\cos^2 2\beta}\right)$$
$$m_{H^+}^2 = M_A^2 + M_W^2, \qquad (15)$$

and the mixing angle α in the (H^0, h^0)-system is derived from

$$\tan 2\alpha = \tan 2\beta\,\frac{M_A^2+M_Z^2}{M_A^2-M_Z^2}\,, -\frac{\pi}{2} < \alpha \le 0\,. \qquad (16)$$

Hence, masses and couplings are determined by only a single parameter more than in the standard model.

The dependence on M_A is symmetric under $\tan\beta \leftrightarrow 1/\tan\beta$, and m_{h^0} is constrained by:

$$m_{h^0} < M_Z\cos 2\beta < M_Z. \qquad (17)$$

This simple scenario, however, is changed when radiative corrections are taken into account.

The tree-level mass matrix m_0 of the neutral scalar system that represents bare mass system is diagonalized by Eqs (12). Loop contributions to the quadratic part of the potential (neglecting the q^2-dependence of the diagrams) modify the mass matrix as

$$m_0 \to m_0 + \delta m = m. \qquad (18)$$

Re-diagonalizing the one-loop matrix m yields the corrected mass eigenvalues m_{H^0,h^0}, replacing (15), and an effective mixing angle α_{eff} instead of (16). The renormalization constants (Dabelstein, 1995) are defined as follows:

$$B_\mu \to (Z_2^B)^{1/2}B_\mu\,, \qquad W_\mu^a \to (Z_2^W)^{1/2}W_\mu^a\,,$$
$$H_i \to Z_{H_i}^{1/2}H_i\,,$$
$$\psi_j^L \to \left(Z_L^j\right)^{1/2}\psi_j^L\,, \qquad \psi_{j\sigma}^R \to \left(Z_R^{j\sigma}\right)^{1/2}\psi_{j\sigma}^R\,,$$
$$g_2 \to Z_1^W(Z_2^W)^{-3/2}g_2, \quad g_1 \to Z_1^B(Z_2^B)^{-3/2}g_1\,,$$
$$v_i \to Z_{H_i}^{1/2}(v_i - \delta v_i)\,,$$
$$m_i^2 \to Z_{H_i}^{-1}(m_i^2 + \delta m_i^2),$$
$$m_{12}^2 \to Z_{H_1}^{-1/2}Z_{H_2}^{-1/2}(m_{12}^2 + \delta m_{12}^2). \qquad (19)$$

The complete definitions and the explicit expressions of the renormalization constants of the other sectors: sfermion sector, MSSM parameters and fields including those of SM as the electric charge and the masses of W, Z, and the fermions and their counterterms in addition to tan β, all these are treated as described in (Fritzsche and Hollik, 2004; Hollik and Rzehak, 2003; Denner, 1993), to deliver all counterterms required for propagators and vertices appearing in the amplitudes.

Real Photonic Corrections

The soft (IR) divergences in the $\mathcal{M}_{\text{virtual}}$ originate from the contributions of virtual photon exchange in loops (Hooft and Veltman, 2007). Due to the selectron exchange channels, one cannot separate off all Feynman diagrams with an additional photon attached to the tree-level diagrams to define pure “weak and QED corrections”. These soft (IR) divergencies can be cancelled by the real photon bremsstrahlung corrections in the soft photon limit in which the cross section for real photon emission (Giele and Glover, 1992) is proportional to the Born cross section,

$$\left(\frac{d\sigma}{d\Omega}\right)_{soft} = \left(\frac{d\sigma}{d\Omega}\right)_{Born} \Delta_{soft} \qquad (20)$$

where Δ_{soft} is the QED correction factor from real bremsstrahlung in the soft-photon approximation. The real photonic corrections are induced by the process:

$$e^+(p_1) + e^-(p_2) \to \tilde{\chi}_1^0(p_3) + \tilde{\chi}_1^0(p_4) + h^0(p_5) + \gamma(k_\gamma),$$

where k_γ denotes the photon momentum.

A real photon radiates from the electron/positron $e^\pm$, and can have either soft or collinear nature. The collinear singularity is regularized by keeping electron (positron) mass. The general phase-space-slicing method (PSS) is adopted to separate the soft photon emission singularity from the real photon emission processes. By using this method, the bremsstrahlung phase space is divided into singular and non-singular regions by the soft photon cut off, ΔE, and the cross section of the real photon emission process is decomposed into soft and hard terms (Al-Negashi *et al*., 2013) as:

$$\boldsymbol{\sigma^{real}} = \boldsymbol{\sigma^{soft}(\Delta E)} + \boldsymbol{\sigma^{hard}(\Delta E)}$$

$$= \sigma^0(\Delta_{soft} + \Delta_{hard}) \qquad (21)$$

where $\sigma^{weak} = \sigma^{soft}$ and $\sigma^{QED} = \sigma^{hard}$.

The energy of the radiated photon in the center of mass system frame is considered as a soft term, Δ_{soft}, with radiated photon energy $k_\gamma^0 < \Delta E$, and a hard term, Δ_{hard}, with $k_\gamma^0 > \Delta E$, where:

$$k_\gamma^0 = \sqrt{\left|\vec{k}_\gamma\right|^2 + m_\gamma^2}$$

and m_γ is the photon mass, which is used to regulate the (IR) divergences existing in the soft term. For practical calculations, σ^{hard} is divided into a collinear part, where the photon is within an angle smaller than $\Delta\theta$ with respect to the radiating particles, and the complementary non-collinear part (Fritzsche and Hollik, 2004),

$$\sigma^{hard} = \sigma^{coll}(\Delta\theta) + \sigma^{non-coll}(\Delta\theta) \qquad (22)$$

$\sigma^{non-coll}$ is calculated numerically with the help of multidimensional numerical integration routines DIVONNE that based on Monte Carlo, and CUHRE ,which are both part of the CUBA-library (Hahn, 2005). In Eq. (21), ΔE depends largely on the weak and QED components $\propto \log\frac{\Delta E^2}{s}$. Therefore, we extract the ΔE terms and the leading logarithms $L_e \equiv log\left(\frac{s}{m_e^2}\right)$, caused by collinear soft photon emission, from the weak corrections and add them to the QED corrections such that both corrections are now cutoff independent (Denner and Dittmaier, 1993). The main part of the QED corrections arises from these leading logarithms L_e, resulting from photons in the beam direction. This leads to a large dependence on the experimental cuts and detector specifications. Therefore, We use the structure function formalism (Denner and Dittmaier, 1993) and subtract the leading logarithmic $\mathcal{O}(\alpha)$ terms of the initial state radiation, $\sigma^{ISR,LL}$, such that only the non-universal QED corrections remain. From the above discussion, we can state the final expression for the total renormalized cross section σ^{total} as following:

$$\sigma^{total} = \sigma^0 + \sigma^{virt} + \sigma^{weak} + \sigma^{QED}, \qquad (23)$$

$$\sigma^{weak} = \sigma^{soft} - \tilde{\sigma}$$

$$\sigma^{QED} = \sigma^{hard} + \tilde{\sigma} - \sigma^{ISR,LL}$$

with

$$\tilde{\sigma} = \frac{\alpha}{\pi}\left[(L_e - 1)log\frac{4\Delta E^2}{s} + \frac{3}{2}L_e\right].\sigma^0 ,$$

$$\sigma^{ISR,LL} = \frac{\alpha}{\pi}L_e\int_0^1 dx\,\Phi(x)\sigma^0(sx),$$

$$\Phi(x) = \lim_{\epsilon\to 0}\left\{\delta(1-x)\left[\frac{3}{2} + 2\log(\epsilon)\right] + \theta(1-x-\epsilon)\frac{1+x^2}{1-x}\right\}$$

The integrated cross section at the one-loop level, can be written in the following way:

$$\sigma^{total} = \sigma^0 + \sigma^0\Delta , \qquad (24)$$

pointing out the relative correction

$$\Delta = (\sigma^{total} - \sigma^0)/\sigma^0, \qquad (25)$$

with respect to the Born cross section.
The relative correction Δ can be decomposed into the following parts, indicating their origin,

$$\Delta = \Delta_{self} + \Delta_{vertex} + \Delta_{box} + \Delta_{QED} + \Delta_{weak} \qquad (26)$$

Numerical Results

In present work, two different scenarios are studied. In the Higgsino scenario the neutralinos are both nearly pure higgsinos and therefore the process is dominated by the s-channel Z_0 exchange. In the Gaugino scenario with binos as $\tilde{\chi}_1^0$ states, the selectron exchange diagrams play the most important role. Both scenarios are chosen such that, for $\tan\beta \approx 10$, the lightest-scalar mass is about $m_h \approx 125$ GeV. It was found from calculations for both scenarios that the SUSY scalar fermions $\tilde{f}$ have masses of order of 1.3 TeV. For the SM input parameters the following values have been used:

$\alpha(m_Z)$	1/127.922
m_W	80.399 GeV
m_Z	91.187 GeV
m_t	174.3 GeV
m_b	4.7 GeV

M_2 is fixed by the gaugino unification relation:

$$M_2 = \frac{5}{3}\tan^2\theta_W M_1,$$

and the gluino mass is related to $M_1$by:

$$m_{\tilde{g}} = (\alpha_s(m_{\tilde{g}})/\alpha)\sin^2\theta_W M_1.$$

where

$$\sin{}^2\theta_W = 1 - m_W^2/m_Z^2$$

In the following numerical examples, the mass spectrum of the SUSY particles are set as shown in Tables 1 and 2 for

Higgsino and Gaugino scenarios respectively. The free parameters that have been used in the calculations are specified as follows:

- For simplicity, all soft-SUSY-breaking parameters are assumed equal and all trilinear couplings are set to a common value A in the sfermions sector.
- The mixing between sfermion generations is neglected, such that, $M_{SUSY} \equiv \widetilde{M}_L \simeq \widetilde{M}_R$.
- The MSSM Higgs sector is parametrized by the CP-odd mass, m_A, and $\tan\beta$, taking into account radiative corrections with the help of FormCalc.
- The chargino–neutralino sector is fixed by choosing a value for the gaugino-mass terms M_1, M_2 and for the Higgsino-mass term μ.

Table 1. The mass spectrum of the SUSY particles for Higgsino scenario

Particle	Mass/(GeV)	Particle	Mass/(GeV)
h^0	125.049	$\tilde{\chi}_1^0$	86.3240
H^0	700.275	$\tilde{\chi}_2^0$	111.646
A^0	700.000	$\tilde{\chi}_3^0$	200.218
$H^{\pm}$	704.600	$\tilde{\chi}_4^0$	416.025
$\tilde{g}$	1063.46	$\tilde{\nu}_e$	1323.46
$\tilde{\chi}_1^{\pm}$	99.2230	$\tilde{\nu}_\mu$	1323.46
$\tilde{\chi}_2^{\pm}$	416.032	$\tilde{\nu}_\tau$	1323.46
$\tilde{e}_L$	1325.68	$\tilde{e}_R$	1325.85
$\tilde{\mu}_L$	1325.67	$\tilde{\mu}_R$	1325.87
$\tilde{\tau}_L$	1324.83	$\tilde{\tau}_R$	1326.71
$\tilde{u}_L$	1323.92	$\tilde{u}_R$	1324.54
$\tilde{d}_L$	1325.23	$\tilde{d}_R$	1326.31
$\tilde{c}_L$	1323.86	$\tilde{c}_R$	1324.60
$\tilde{s}_L$	1325.26	$\tilde{s}_R$	1326.31
$\tilde{t}_L$	1308.68	$\tilde{t}_R$	1361.51
$\tilde{b}_L$	1323.25	$\tilde{b}_R$	1328.29

Table 2. The mass spectrum of the SUSY particles for Gaugino scenario

Particle	Mass/(GeV)	Particle	Mass/(GeV)
h^0	125.116	$\tilde{\chi}_1^0$	91.5340
H^0	394.039	$\tilde{\chi}_2^0$	181.009
A^0	393.600	$\tilde{\chi}_3^0$	359.502
$H^{\pm}$	401.727	$\tilde{\chi}_4^0$	378.874
$\tilde{g}$	525.351	$\tilde{\nu}_e$	1298.43
$\tilde{\chi}_1^{\pm}$	180.516	$\tilde{\nu}_\mu$	1298.43
$\tilde{\chi}_2^{\pm}$	379.562	$\tilde{\nu}_\tau$	1298.43
$\tilde{e}_L$	1300.69	$\tilde{e}_R$	1300.87
$\tilde{\mu}_L$	1300.61	$\tilde{\mu}_R$	1300.95
$\tilde{\tau}_L$	1298.25	$\tilde{\tau}_R$	1303.31
$\tilde{u}_L$	1298.89	$\tilde{u}_R$	1299.53
$\tilde{d}_L$	1300.23	$\tilde{d}_R$	1301.34
$\tilde{c}_L$	1298.89	$\tilde{c}_R$	1299.54
$\tilde{s}_L$	1300.21	$\tilde{s}_R$	1301.35
$\tilde{t}_L$	1301.68	$\tilde{t}_R$	1319.36
$\tilde{b}_L$	1294.11	$\tilde{b}_R$	1307.44

1. Higgsino Scenario

The parameters are set as:

$\{M_2, \mu, A, \tan\beta, m_A, , M_{SUSY}\}$ = {400 GeV, -100 GeV, 400 GeV, 10, 700 GeV, 1325 GeV}

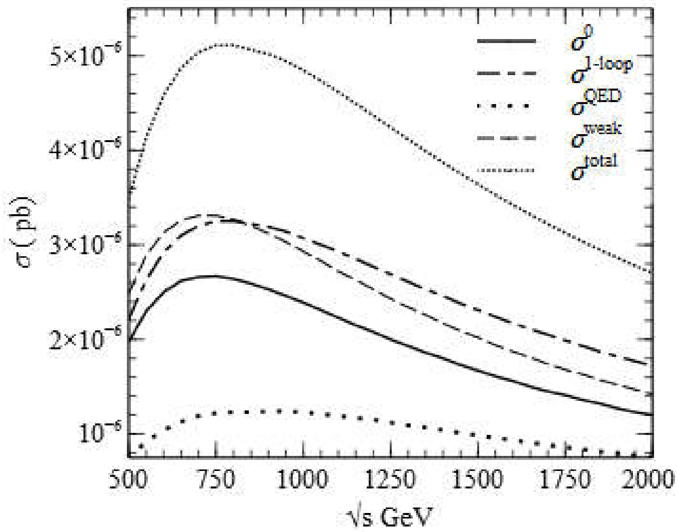

Figure 5. Total cross section as a function of $\sqrt{s}$ in the Higgsino scenario

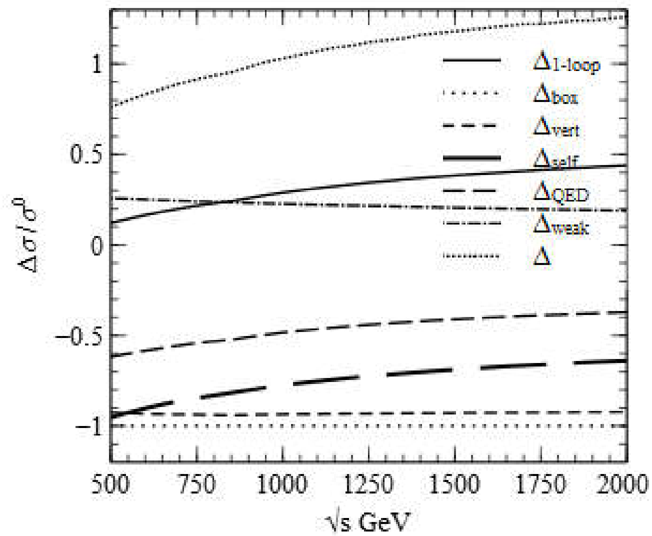

Figure 6. Relative corrections as a function of $\sqrt{s}$ in the Higgsino scenario

Table 3. The maximum cross sections in Higgsino scenario

	$(\sigma)_{max}$ /Pb	$\sqrt{s}$ /GeV
Born	2.67×10^{-6}	750
1-loop	3.25×10^{-6}	775
QED	1.24×10^{-6}	950
Weak	3.31×10^{-6}	750
Total	5.11×10^{-6}	775

2. Gaugino Scenario

The parameters are set as:

$\{M_2, \mu, A, \tan\beta, m_A, , M_{SUSY}\}$ = {197.6 GeV, 353.1 GeV, –100 GeV, 10.2, 393.6 GeV, 1300 GeV}

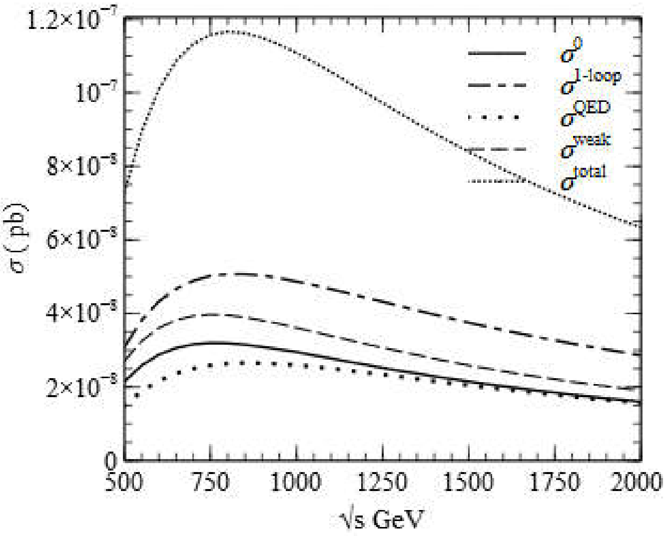

Figure 7. Total cross section as a function of $\sqrt{s}$ in the Gaugino scenario

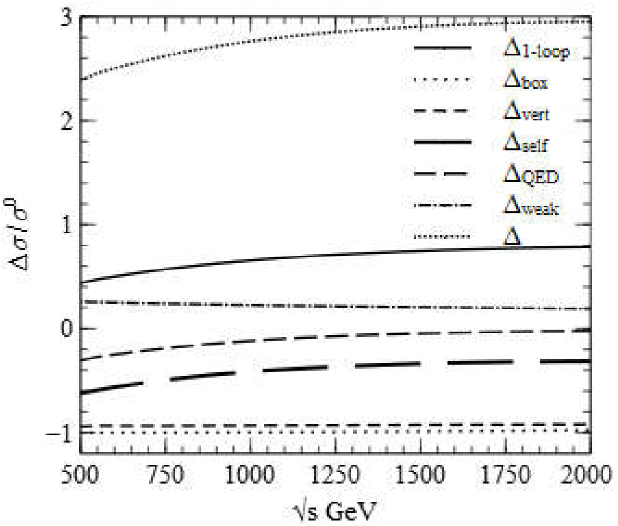

Figure 8. Relative corrections as a function of $\sqrt{s}$ in the Gaugino scenario

Table 4. The maximum cross sections in Gaugino scenario

	$(\sigma)_{max}$ /Pb	$\sqrt{s}$ /GeV
Born	3.20×10^{-8}	750
1-loop	5.07×10^{-8}	820
QED	2.66×10^{-8}	850
Weak	3.97×10^{-8}	750
Total	1.17×10^{-7}	800

Conclusion

In this work, the full electroweak radiative corrections at one-loop level to the lightest neutralino pair production with light neutral Higgs boson at electron-positron LC have been calculated in the frame of MSSM. The calculation was performed in an analytical using the FeynArts-3.6 and FormCalc-7.1 computer packages, where we modified the MSSM model file implemented in FeynArts-3.6. We have calculated the weak and QED corrections, which contribute significantly to the total cross section. The full electroweak radiative corrections have improved the LO cross section by about 76-125% for Higgsino scenario, and by about 239-296% for Gaugino scenario, thus they have to be taken into account in future linear collider experiments and in the theoretical calculations. The maximum cross sections are presented in Tables 3 and 4 for both scenarios.

The same process had been studied for the same parameters for each scenario but with different values of M_{SUSY} (Seif, 2017). The comparison between the results in this work and those in Ref. (Seif, 2017) shows the effect of M_{SUSY} value on the lightest MSSM CP-even Higgs particle mass as shown in the following table:

	Higgsino scenario		Gaugino scenario	
	Previous study(28)	Current study	Previous study(28)	Current study
M_{SUSY}(GeV)	350	1325	500	1300
m_h (GeV)	105.341	125.049	110.985	125.116

Since the mass of the Higgs boson has been confirmed by experimental investigation to be in a range of 125-127 GeV, It is expected that the mass scale of superpartner to be in the range of 1-2 TeV. In general, by comparing the cross section values of the two scenarios, it is found that the Higgsino scenario has larger values for all types of correction than that of the Gaugino scenario.

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