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# RESEARCH ARTICLE

## INTERFACE EFFECTS ON AN INFINITE PLANE CONTAINING TWO CIRCULAR NANO-INHOMOGENEITIES

**Lei Dongxia, *Wen Shuangshuang and Ou Zhiying**

School of Science, Lanzhou University of Technology, Lanzhou 730050, P. R. China

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## ABSTRACT

Based on the surface theory, employing complex variable function method, the problem of an isotropic elastic plane containing two circular nano-inhomogeneities is studied. The solutions of stress field of any point at the nano- inhomogeneities and plane with interface effects are obtained. The surface effects on the stress fields of the whole plane structure are analyzed.

***Key words:*** Nano-inhomogeneities; Complex Variable Function Method, Surface Theory

## INTRODUCTION

With the development of science and technology, the study of the mechanical behavior of inhomogeneous materials containing nanosized inclusions or holes has attracted great interest. The existing literatures show that researches with interface/surface effects of an inclusion or a hole on mechanical behavior of inhomogeneous medium have achieved some achievements (Wang, 2007; Ou, 2008; Grekov, 2014; Ou et al., 2009; Tian, 2007; Tian, 2007; Ou, 2015; Mogilevskaya, 2008; Ou, 2015), but the lack of studies of surface/interface effects of multiple holes or inclusions on inhomogeneous media. Gong and Meguid, (1993) use the complex variable function method to analyze the interacting circular inhomogeneities in plane elastostatics without the surface stress. Based on the displacement Gurtin and Murdoch models, Mogilevskaya and Crouch (2008), the boundary integral method are used to analyze the two dimensional problems with multiple nanosized circular holes. The purpose of the paper is to analyze the effects of surface on the infinite plane containing two circular nano-inhomogeneities subjected to stress and shear stress at infinity. Based on the surface theory, complex variable function method is used here to solve the deformation field.

### Mathematical Formulations

Consider an isotropic elastic plane $M$ containing two circular nano-inhomogeneities $O_1$ and $O_2$ as shown in Fig. 1. It is subjected to stress and shear stress at infinity of the plane. With the complex variable method, the stress and the displacement can be expressed in terms of two analytic functions $\varphi(z)$ and $\psi(z)$ as follows

$$2\mu(u_x + iu_y) = k\varphi(z) - z\overline{\varphi'(z)} - \overline{\psi(z)} \tag{1}$$

$$\sigma_{xx} + \sigma_{yy} = 2\left(\varphi'(z) + \overline{\varphi'(z)}\right) \tag{2}$$

$$\sigma_{yy} - \sigma_{xx} + 2i\sigma_{xy} = 2\left(\bar{z}\varphi''(z) + \psi'(z)\right) \tag{3}$$

where $u_{ij}$ and $\sigma_{ij}$ are displacement and stress tensor, respectively; $k = 3 - 4\upsilon$ for plane strain and $k = (3-\upsilon)/(1+\upsilon)$ for plane stress; μ and υ are the shear modulus and Poisson's ratio, respectively; $z = x + iy = ae^{i\theta}$. Airy's stress function U can be related to the above two functions by

$$U = \mathrm{Re}\left(\bar{z}\varphi(z) + \eta(z)\right), \ \eta'(z) = \psi(z) \tag{4}$$

****Corresponding author: Wen Shuangshuang ,***
*School of Science, Lanzhou University of Technology, Lanzhou 730050, P. R. China.*

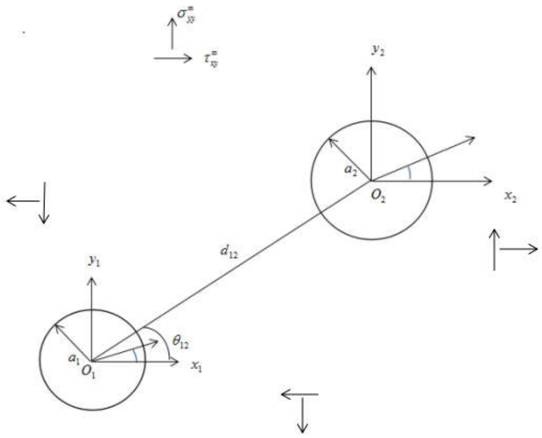

**Fig. 1. Elastic plane containing two circular nano-inhomogeneities**

The components of stress and displacement in terms of polar coordinates are

$$2\mu(u_r + iu_\theta) = e^{-i\theta}\left(k\varphi(z) - z\overline{\varphi'(z)} - \overline{\psi(z)}\right) \tag{5}$$

$$\sigma_{rr} + \sigma_{\theta\theta} = 2\left(\varphi'(z) + \overline{\varphi'(z)}\right) \tag{6}$$

$$\sigma_{\theta\theta} - \sigma_{rr} + 2i\sigma_{r\theta} = 2\left(\bar{z}\varphi''(z) + \psi'(z)\right)e^{2i\theta} \tag{7}$$

Equations (6) and (7) can be expressed in the following complex variable form

$$\sigma_{rr} - i\sigma_{r\theta} = \varphi'(z) + \overline{\varphi'(z)} - \left(\bar{z}\varphi''(z) + \psi'(z)\right)e^{2i\theta} \tag{8}$$

In the bulk, the constitutive equations are

$$\sigma_{ij}^B = \lambda\varepsilon_{kk}\delta_{ij} + 2\mu\varepsilon_{ij} \tag{9}$$

In the $(r,\theta,t)$ coordinate, the Young-Laplace equations are

$$\nabla_s \cdot \tau = \left(\frac{1}{r}\frac{\partial\sigma_{\theta t}^s}{\partial\theta} + \frac{\partial\sigma_{tt}^s}{\partial t}\right)e_t + \left(\frac{1}{r}\frac{\partial\sigma_{\theta\theta}^s}{\partial\theta} + \frac{\partial\sigma_{t\theta}^s}{\partial t}\right)e_\theta + \left(-\frac{\partial\sigma_{\theta\theta}^s}{r}\right)n \tag{10}$$

$$[\![\sigma_{rt}^B]\!]e_t + [\![\sigma_{r\theta}^B]\!]e_\theta + [\![\sigma_{rr}^B]\!]n = -\nabla_s \cdot \tau \tag{11}$$

$$\sigma_{\alpha\beta}^s = \tau^0\delta_{\alpha\beta} + 2\left(\mu^s - \tau^0\right)\varepsilon_{\alpha\gamma}\delta_{\gamma\beta} + \left(\lambda^s + \tau^0\right)\varepsilon_{\gamma\gamma}\delta_{\alpha\beta} \tag{12}$$

where superscripts $B$ and $S$ are used to denote the quantities corresponding to bulk and interface between matrix and inhomogeneity; $\lambda$ and $\mu$ are the Lame constants; $n$ is the normal vector on the interface; $\tau^0$ is the residual surface stress under unstrained conditions; $\delta_{ij}$ is the Kronecker delta; $[\![*]\!] = (*)_M - (*)_I$ denotes the jump across the matrix-inhomogeneity interface; $i$, $j$, $k = 1$, $2$, $3$; $\alpha$, $\beta$, $\gamma = 1$, $2$.

Assume that the inhomogeneities are perfectly bonded to the matrix. Then the displacements are continuous at the interface

$$\left(u_r + iu_\theta\right)_M = \left(u_r + iu_\theta\right)_I + \left(u_r + iu_\theta\right)_I^* \tag{13}$$

where the last term is the displacement induced by the prescribed uniform dilatational eigenstrain $\varepsilon^*$, i.e., $\varepsilon_{11}^* = \varepsilon_{22}^* = \varepsilon^*$, and

$$(u_r + iu_\theta)_I^* = a_{0,1}\varepsilon^* \text{ at } a_1 = a_{0,1} \tag{14}$$

Here, $z_1 = a_1 e^{i\theta_1}$.

From equations (10) and (11), we have

$$[\![\sigma_{rt}^B]\!] = -\left(\frac{1}{a_{0,1}}\frac{\partial \sigma_{\theta t}^s}{\partial \theta} + \frac{\partial \sigma_{tt}^s}{\partial t}\right) \tag{15}$$

$$[\![\sigma_{r\theta}^B]\!] = -\left(\frac{1}{a_{0,1}}\frac{\partial \sigma_{\theta\theta}^s}{\partial \theta} + \frac{\partial \sigma_{\theta t}^s}{\partial t}\right) \tag{16}$$

$$[\![\sigma_{rr}^B]\!] = \frac{\sigma_{\theta\theta}^s}{a_{0,1}} \tag{17}$$

For plane problems, the quantities with respect to $t$ are 0. Thus equation (15) is automatically satisfied. Equations (16) and (17) can be expressed

$$[\![\sigma_{rr}^B - i\sigma_{r\theta}^B]\!] = \frac{\sigma_{\theta\theta}^s}{a_{0,1}} + \frac{i}{a_{0,1}}\frac{\partial \sigma_{\theta\theta}^s}{\partial \theta} \tag{18}$$

The left-hand side of equation (18) can be written in terms of potential functions by using equation (8). For the right-hand side, the surface stress $\sigma_{\theta\theta}^s$ by using equation (12) can be written as

$$\sigma_{\theta\theta}^s = \tau^0 + 2(\mu^s - \tau^0)\varepsilon_{\theta\theta} + (\lambda^s + \tau^0)(\varepsilon_{tt} + \varepsilon_{\theta\theta}) \tag{19}$$

When $a_1 = a_{0,1}$, the elastic strain by using equations (6), (7) and (9) can be obtained from the following equations

$$\varepsilon_{rr} + \varepsilon_{\theta\theta} = \frac{1}{\Gamma}\left(\varphi'(z) + \overline{\varphi'(z)}\right) \tag{20}$$

$$\varepsilon_{\theta\theta} - \varepsilon_{rr} + 2i\varepsilon_{r\theta} = \frac{1}{\mu}\left(\bar{z}\varphi''(z) + \psi'(z)\right)e^{2i\theta} \tag{21}$$

Here, $\Gamma = \mu + \lambda$. Therefore

$$\varepsilon_{\theta\theta} = \frac{1}{2\Gamma}\left(\varphi'(z) + \overline{\varphi'(z)}\right) + \frac{1}{4\mu}\left(\bar{z}\varphi''(z) + \psi'(z)\right)e^{2i\theta} + \frac{1}{4\mu}\left(z\overline{\varphi''(z)} + \overline{\psi'(z)}\right)e^{-2i\theta} \tag{22}$$

The strain $\varepsilon_{\theta\theta}$ is continuous at the interface due to the continuous displacement at the interface. Thus in the following derivation, the strain is calculated from the matrix. Note that $\varepsilon_{tt} = 0$ for plane strain and $\varepsilon_{tt} = \frac{v}{v-1}(\varepsilon_{rr} + \varepsilon_{\theta\theta})$ with $a_1 = a_{0,1}$ for plane stress. Because of the discontinuity of $\varepsilon_{tt}$ at the interface, the mean strain is used

$$\varepsilon_{tt} = \frac{1}{2}\left((\varepsilon_{tt})_M + (\varepsilon_{tt})_I\right) \tag{23}$$

$(\varepsilon_{tt})_M$ can be obtained from equation (20), however, $(\varepsilon_{tt})_I$ is

$$(\varepsilon_{tt})_I = \frac{v_I}{v_I - 1}\left((\varepsilon_{rr} + \varepsilon_{\theta\theta})_I^e + 2\varepsilon^*\right) \tag{24}$$

where $(\varepsilon_{rr} + \varepsilon_{\theta\theta})_I^e$ can be obtained by using equation (20).

Since no singularities are assumed to reside inside or on the boundary of the inhomogeneity, $\varphi(z)$ and $\psi(z)$ can be expanded into Laurent series as follows

$$\varphi_I(z_1) = \sum_{n=1}^{\infty} H_{n,1} z_1^n \tag{25a}$$

$$\psi_I(z_1) = \sum_{n=1}^{\infty} L_{n,1} z_1^n \tag{25b}$$

However, if there are one inhomogeneity in the matrix $M$ (Tianet al., 2007), $\varphi(z)$ and $\psi(z)$ can be expanded into Laurent series as follows

$$\varphi_M(z_1) = Az_1 + \sum_{n=1}^{\infty} F_{n,1} z_1^{-n} \tag{26a}$$

$$\psi_M(z_1) = Bz_1 + \sum_{n=1}^{\infty} B_{n,1} z_1^{-n} \tag{26b}$$

When there are two inhomogeneity in the matrix $M$, the Airy's stress function can be expanded by [Gong et al.,1993] as follows

$$U = U_0 + \sum_{k=1}^{2} U_k \tag{27}$$

where $U_0$ is the stress function corresponding to the uniform stress state at infinity in the absence of the inhomogeneity and the stress function $U_k$ is contains singularities inside the k-th inhomogeneity. So $U_0$ can be expanded as

$$U_0 = \mathrm{Re}\left(\bar{z}_1 \varphi_0(z_1) + \eta_0(z_1)\right), \quad \eta_0'(z_1) = \psi_0(z_1) \tag{28a}$$

In the view of equations (2), (3), (26) and England (1971), we have

$$\varphi_0(z_1) = \frac{\sigma_{xx}^{\infty} + \sigma_{yy}^{\infty}}{4} z_1 = Az_1 \tag{28b}$$

$$\psi_0(z_1) = \frac{\sigma_{yy}^{\infty} - \sigma_{xx}^{\infty} + 2i\sigma_{xy}^{\infty}}{2} z_1 = Bz_1 \tag{28c}$$

where $\sigma_{yy}^{\infty}$, $\sigma_{xx}^{\infty}$ and $\sigma_{xy}^{\infty}$ are the far-field stresses which in the case of our study are assumed to be $\sigma_{yy}^{\infty} = \sigma_0$, $\sigma_{xx}^{\infty} = c\sigma_0$, $\sigma_{xy}^{\infty} = 0$,

$$\frac{\sigma_{xx}^{\infty}}{\sigma_{yy}^{\infty}} = c\,(\text{real number}) \begin{cases} c = 1 & \text{(biaxial loadings)} \\ c = 0 & \text{(uniaxial loadings)} \\ c = -1 & \text{(shear loadings)} \end{cases}.$$

$U_k$ can be expanded as

$$U_k = \mathrm{Re}\left(\overline{z_k}\hat{\varphi}(z_k) + \hat{\eta}(z_k)\right), \quad \hat{\eta}'(z_k) = \hat{\psi}(z_k) \tag{29a}$$

$$\hat{\varphi}(z_k) = \sum_{n=1}^{\infty} F_{n,k} z_k^{-n} \tag{29b}$$

$$\hat{\psi}(z_k) = \sum_{n=1}^{\infty} B_{n,k} z_k^{-n} \tag{29c}$$

$$z_2 = z_1 - d_{12} e^{i\theta_{12}} \tag{30}$$

Substituting equations (28), (29), (30) into equation (27), it can be reduced to the same form as equation (4),

$$\varphi_M(z_1) = Az_1 + \sum_{n=1}^{\infty} F_{n,1} z_1^{-n} + \sum_{n=1}^{\infty} M_{n,1} z_1^{n} \tag{31a}$$

$$\psi_M(z_1) = Bz_1 + \sum_{n=1}^{\infty} B_{n,1} z_1^{-n} + \sum_{n=1}^{\infty} K_{n,1} z_1^{n} \tag{31b}$$

Here,

$$M_{n,1} = \sum_{p=1}^{\infty} C_{n+p-1}^{p-1} F_{p,2} (-1)^p \frac{1}{d_{12}^{n+p}} e^{-(n+p)i\theta_{12}},$$

$$K_{n,1} = \sum_{p=1}^{\infty} C_{n+p-1}^{p-1} B_{p,2} (-1)^p \frac{1}{d_{12}^{n+p}} e^{-(n+p)i\theta_{12}} - \sum_{p=1}^{\infty} C_{n+p-1}^{p-1} F_{p,2} (-1)^p \frac{1}{d_{12}^{n+p-1}} e^{-(n+p+1)i\theta_{12}}.$$

In the same way, the displacement and stress boundary condition of $O_2$ can be respectively expressed as

$$(u_r + iu_\theta)_M = (u_r + iu_\theta)_I + a_{0,2}\varepsilon^* \tag{32}$$

$$[\![\sigma_{rr}^B - i\sigma_{r\theta}^B]\!] = \frac{\sigma_{\theta\theta}^s}{a_{0,2}} + \frac{i}{a_{0,2}} \frac{\partial \sigma_{\theta\theta}^s}{\partial \theta} \tag{33}$$

$\varphi(z)$ and $\psi(z)$ can be expanded into Laurent series as

$$\varphi_I(z_2) = \sum_{n=1}^{\infty} H_{n,2} z_2^{n} \tag{34a}$$

$$\psi_I(z_2) = \sum_{n=1}^{\infty} L_{n,2} z_2^{n} \tag{34b}$$

$$\varphi_M(z_2) = Az_2 + \sum_{n=1}^{\infty} F_{n,2} z_2^{-n} + \sum_{n=1}^{\infty} M_{n,2} z_2^{n} \tag{35a}$$

$$\psi_M(z_2) = Bz_2 + \sum_{n=1}^{\infty} B_{n,2} z_2^{-n} + \sum_{n=1}^{\infty} K_{n,2} z_2^{n} \tag{35b}$$

Here,

$$M_{n,2} = \sum_{p=1}^{\infty} C_{n+p-1}^{p-1} F_{p,1} (-1)^p \frac{1}{d_{12}^{n+p}} e^{-(n+p)i\theta_{12}},$$

$$K_{n,2} = \sum_{p=1}^{\infty} C_{n+p-1}^{p-1} B_{p,1} (-1)^p \frac{1}{d_{12}^{n+p}} e^{-(n+p)i\theta_{12}} - \sum_{p=1}^{\infty} C_{n+p-1}^{p-1} F_{p,1} (-1)^p \frac{1}{d_{12}^{n+p-1}} e^{-(n+p+1)i\theta_{12}}.$$

Substituting equations (5), (14), (25), (31) into equation (13), which are the displacement of the inclusion $O_1$, it is expressed as follows

$$\frac{K_M}{\mu_M}\left(Az_1 + \sum_{n=1}^{\infty} F_{n,1} z_1^{-n} + \sum_{n=1}^{\infty} M_{n,1} z_1^{n}\right) - \frac{A}{\mu_M} z_1 - \frac{1}{\mu_M} \sum_{n=1}^{\infty} \overline{M}_{n,1} z_1^{-n+2} a_1^{2n-2} n - \frac{B}{\mu_M} z_1^{-1} a_1^2$$

$$-\frac{1}{\mu_M}\sum_{n=1}^{\infty}\overline{B}_{n,1}z_1^n a_1^{-2n}-\frac{1}{\mu_M}\sum_{n=1}^{\infty}\overline{K}_{n,1}z_1^{-n}a_1^{2n}$$
$$+\frac{1}{\mu_M}\sum_{n=1}^{\infty}\overline{F}_{n,1}z_1^{n+2}a_1^{-(2n+2)}n$$
$$=\frac{K_I}{\mu_I}\sum_{n=1}^{\infty}H_{n,1}z_1^n-\frac{1}{\mu_I}\sum_{n=1}^{\infty}\overline{H}_{n,1}z_1^{-n+2}a_1^{2n-2}n$$
$$-\frac{1}{\mu_I}\sum_{n=1}^{\infty}\overline{L}_{n,1}z_1^{-n}a_1^{2n}+\frac{2a_{0,1}\varepsilon^* z_1}{a_1} \quad (36)$$

As to the same the coefficients of $z_1^n$ ,we can obtain the following equations

$$\frac{K_M}{\mu_M}A+\frac{K_M}{\mu_M}M_{1,1}-\frac{A}{\mu_M}-\frac{1}{\mu_M}\overline{M}_{1,1}-\frac{1}{\mu_M}\overline{B}_{1,1}a_1^{-2}$$
$$=\frac{K_I}{\mu_I}H_{1,1}-\frac{1}{\mu_I}\overline{H}_{1,1}+\frac{2a_{0,1}\varepsilon^*}{a_1}$$
$$\frac{1}{\mu_M}\overline{M}_{2,1}a_1^2=\frac{1}{\mu_I}\overline{H}_{2,1}a_1^2$$
$$\frac{K_M}{\mu_M}F_{1,1}-\frac{3}{\mu_M}\overline{M}_{3,1}a_1^4-\frac{B}{\mu_M}a_1^2-\frac{1}{2\mu_M}\overline{K}_{1,1}a_1^2$$
$$=-\frac{3}{\mu_I}\overline{H}_{3,1}a_1^4-\frac{1}{\mu_I}\overline{L}_{1,1}a_1^2$$
$$\frac{K_M}{\mu_M}M_{2,1}-\frac{1}{\mu_M}\overline{B}_{2,1}a_1^{-4}=\frac{K_I}{\mu_I}H_{2,1}$$
$$\frac{K_M}{\mu_M}M_{3,1}+\frac{1}{\mu_M}\overline{F}_{1,1}a_1^{-4}-\frac{1}{\mu_M}\overline{B}_{3,1}a_1^{-6}=\frac{K_I}{\mu_I}H_{3,1} \quad (37)$$

Similarly, we can obtain the coefficients of $z_2^n$ by substituting equations (5), (34), (35) into equation (32), which are the displacement of the inclusion $O_2$. Due to the stress boundary $O_1$ and $O_2$, the coefficients of $z_1^n$ and $z_2^n$ can be calculated by substituting equations (8), (19), (22), (23), (24), (25), (31) into equation (18), and by substituting equations (8), (19), (22), (23), (24), (34), (35) into equation (33).

The solution yields

$$B_{2,2}=B_{2,1}=H_{2,2}=H_{2,1}=M_{2,2}=M_{2,1}=0 .$$

Then the unknown coefficients $F_{1,1}$, $B_{1,1}$, $B_{3,1}$, $H_{1,1}$, $H_{3,1}$, $L_{1,1}$, $F_{1,2}$, $B_{1,2}$, $B_{3,2}$, $H_{1,2}$, $H_{3,2}$, $L_{1,2}$ are determined.

In the matrix $M$ , the stresses are

$$\sigma_{rr}=2\left(A-F_{1,1}a_1^{-2}\cos(2\theta_1)-F_{1,2}\frac{1}{d_{12}}\cos(2\theta_{12})\right)$$
$$-2F_{1,1}a_1^{-2}\cos(2\theta_1)-9F_{3,2}\frac{a_1^2}{d_{12}^{\ 3}}\cos(5\theta_{12}-4\theta_1)$$
$$+3B_{3,1}a_1^{-4}\cos(2\theta_1)+B_{1,2}\frac{1}{d_{12}^{\ 2}}\cos(2\theta_{12}-2\theta_1)$$
$$+9B_{3,2}\frac{a_1^2}{d_{12}^{\ 4}}\cos(4\theta_{12}-4\theta_1)+B_{1,1}a_1^{-2}$$
$$-F_{1,2}\frac{1}{d_{12}^{\ 2}}\cos(3\theta_{12}-2\theta_1)-B\cos(2\theta_1) \quad (38a)$$

$$\sigma_{r\theta} = 9F_{3,2}\frac{a_1^2}{d_{12}^{\ 3}}\sin(5\theta_{12}-4\theta_1)$$
$$-3B_{3,1}a_1^{-4}\sin(2\theta_1) - B_{1,2}\frac{1}{d_{12}^{\ 2}}\sin(2\theta_{12}-2\theta_1)$$
$$+F_{1,2}\frac{1}{d_{12}^{\ 2}}\sin(3\theta_{12}-2\theta_1) + 2F_{1,1}a_1^{-2}\sin(2\theta_1)$$
$$-9B_{3,2}\frac{a_1^2}{d_{12}^{\ 4}}\sin(4\theta_{12}-4\theta_1) - B\sin(2\theta_1) \quad (38b)$$

$$\sigma_{\theta\theta} = 2\left(A - F_{1,1}a_1^{-2}\cos(2\theta_1) - F_{1,2}\frac{1}{d_{12}}\cos(2\theta_{12})\right)$$
$$+2F_{1,1}a_1^{-2}\cos(2\theta_1) + 9F_{3,2}\frac{a_1^2}{d_{12}^{\ 3}}\cos(5\theta_{12}-4\theta_1)$$
$$-3B_{3,1}a_1^{-4}\cos(2\theta_1) - B_{1,2}\frac{1}{d_{12}^{\ 2}}\cos(2\theta_{12}-2\theta_1)$$
$$-9B_{3,2}\frac{a_1^2}{d_{12}^{\ 4}}\cos(4\theta_{12}-4\theta_1) - B_{1,1}a_1^{-2}$$
$$+F_{1,2}\frac{1}{d_{12}^{\ 2}}\cos(3\theta_{12}-2\theta_1) + B\cos(2\theta_1) \quad (38c)$$

In the inhomogeneity $O_1$, the stresses are

$$\sigma_{rr} = 2H_{1,1} - L_{1,1}\cos(2\theta_1) \quad (39a)$$

$$\sigma_{r\theta} = -6a_1^2H_{3,1}\sin(2\theta_1) - L_{1,1}\sin(2\theta_1) \quad (39b)$$

$$\sigma_{\theta\theta} = 2H_{1,1} + 12a_1^2H_{3,1}\cos(2\theta_1) + L_{1,1}\cos(2\theta_1) \quad (39c)$$

**Numerical Results and Discussions**

Based on the above method, the elastic field around the circular inclusions is obtained by numerical simulation. In what follows, we set

$\mu_M = 34.7\,\text{GPa}$, $V_M = 0.3$, $\lambda_M = 52\,\text{GPa}$, $\varepsilon^* = 0.3$, $\mu_I = 17.3\,\text{GPa}$, $\lambda_I = 26\,\text{GPa}$, $V_1 = 0.3$, $\lambda^s = 3.5$, $\mu^s = -6$, $K^s = -8.6$, $\lambda^s = 6.9$, $\mu^s = -0.5$, $K^s = 5.8$, $\tau^0 = 0.1\,\text{N/m}$.

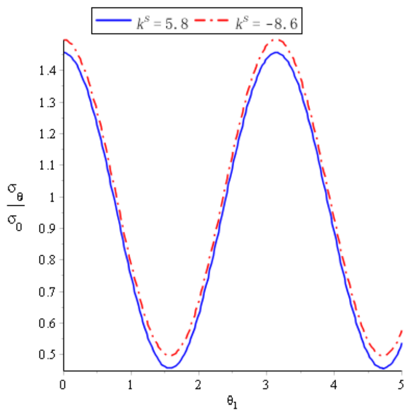


**Fig. 2 The variation of normal hoop stress along $\theta_1$**

$(d_{12} = 1000, \theta_{12} = 0)$

Fig. 2 shows the variation of the normal hoop stress along the $\theta_1$. When the $d_{12}$ tends to infinity and only a circular inclusion effecting on normal stress, there is the same trend of the two curves. Comparing to the Fig. 3 in literature (Tian et al., 2007) [6], it verifies that the result of this paper is the correctness.

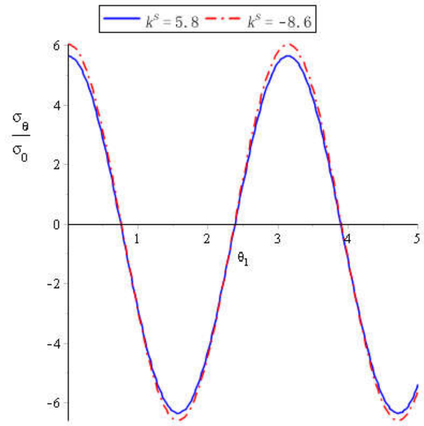


**Fig. 3. The variation of normal hoop stress along $\theta_1$**

$(d_{12} = 0.5, \theta_{12} = 0)$

Fig. 3 shows the variation of the normal hoop stress along the $\theta_1$. Comparing to the Fig. 2, the effect of the inclusion on the normal hoop stress is greater when there are two circular inclusions and the value $d_{12}$ is 0.5.

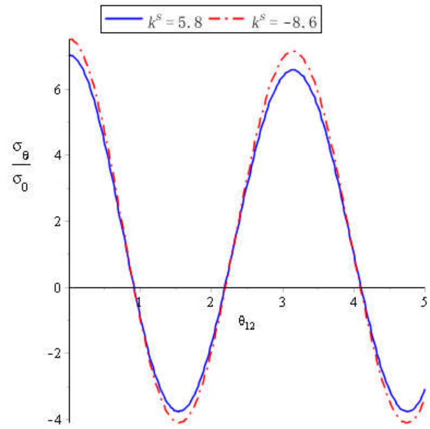


**Fig. 4 The variation of normal hoop stress along $\theta_{12}$**

Fig. 4 shows the effect of $\theta_{12}$ on the normal hoop stress. And the variation of the normal hoop stress along the $\theta_{12}$ is similar to those along the $\theta_1$, but the influence is different

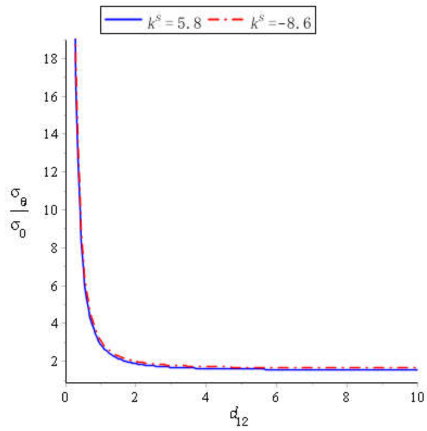


**Fig. 5 The variation of normal hoop stress along $d_{12}$**

The influences of the distance $d_{12}$ between the two circular inclusions on the normal hoop stress are shown in Fig. 5. The stress decreases sharply with the increase of $d_{12}$, and the $d_{12}$ has almost no effect on the normal hoop stress when the values of $d_{12}$ are greater than 2.

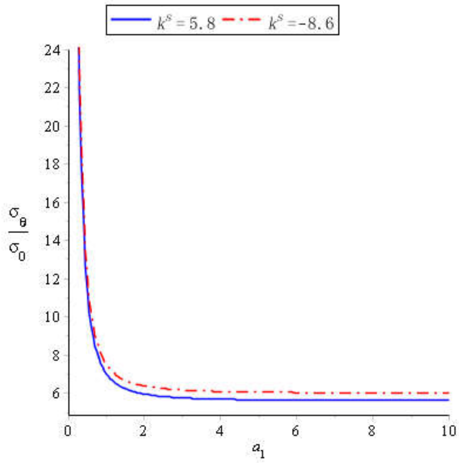


**Fig. 6 The variation of normal hoop stress along $a_1$**

Fig. 6 shows the variation of normal hoop stress along the $a_1$. From Fig. 6, the influence of the radius $a_1$ on the normal hoop stress decreases with the increase of $a_1$, and it has little effect when it is greater than 2.

**Summary and Conclusion**

The influence of the interface effect on the stress field of the composites is demonstrated, which can be helpful for the study of other nanocomposites. It is found that the surface elasticity theory illuminates some interesting characteristics of mechanic at the nanoscale.

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