

RESEARCH ARTICLE

TANGENT OPTIMIZATION OF SKIP-LOT SAMPLING PLAN (SkSP-R) WITH MDS (0, 1) PLAN AS REFERENCE PLAN THROUGH MINIMUM ANGLE CRITERIA

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ABSTRACT

In this paper a skip lot sampling plan of type (SkSP-R) with Multiple Deferred State Sampling plan MDS (0,1) as reference plan has been studied. Under acceptance sampling producer's risk and consumer's risk has become increasingly common in maintaining quality products especially in industries. Producer's risk and Consumer's risk has been minimized by minimizing the tangent angle passing through $(AQL, 1-\alpha)$ and (LQL, β) . Designing methodologies are provided to illustrate the solution procedures.

Key words: Skip-Lot Sampling Plan, Multiple Deferred State Sampling Plan, Minimum Angle Method, Acceptance Quality Level, Limiting Quality Level

INTRODUCTION

Acceptance sampling is an important tool of statistical quality control. This tool is used to enhance the quality of the product through the inspection from the raw stage to the final stage. Without the proper inspection or testing the product may cause the bad reputation of the company in the global market. Good products sent to the market after the inspection increase the demand and alternately increase the profit of the company. Therefore, sampling plans have received the attention of the industrial engineers. The primary objective of sampling inspection is to reduce the cost of inspection while at the same time assuring the customer to satisfy an adequate level of quality on items being inspected. Inspection of raw materials, semi finished products, or a finished product is an important part of quality assurance. When inspection is done for the purpose of acceptance or rejection of a product, and it is based on adherence to a standard the type of inspection procedure employed, such a procedure is usually called acceptance sampling. Sampling is widely used in government sector and industry for controlling the quality of shipment of components, supplies and final products. In this paper a skiplot sampling plan of type SkSP-R with Multiple Deferring State Sampling plan (0,1) as reference plan has been proposed. Producer risk and Consumer Risk has been minimized through minimizing the tangent angle passing through $(AQL, 1-\alpha)$ and (LQL, β) . It is discussed how the declination angle of the tangent at the inflection point of the OC curve which discriminates the Multiple Deferred State Sampling plan. Tables are presented for the selection of plans based on Acceptable Quality Level (AQL) and Limiting Quality Level (LQL) with discriminant or declination angle of the tangent. Dodge (1955) has introduced

the concept of skip-lot sampling, by applying the principles of a continuous sampling plan of type CSP-1 to a series of lots or batches of material. This plan is designated as the SkSP-1 plan and specifically applicable for bulk materials or products produced in successive lots. Perry (1970) has developed a system of sampling inspection plan known as SkSP-2. This Plan involves inspection of only a fraction 'f' of the submitted lots when quality of the submitted product is good as demonstrated by the quality of the product. Peach and Littaur (1946) have considered two points on the OC curve as $(p1, 1-\alpha)$, and $(p2, \beta)$ and propose another method which minimizes the angle between them. Normal Bush *et al.* (1953) have suggested two points on the OC curve namely $(AQL, 1-\alpha)$, and $(IQL, 0.50)$, and the cosine angle of chord length to describe the direction of OC curve. Suresh (1993) has given for the selection of Skip-lot Sampling Plan of type SkSP-2 with reference plans $SSP(c=0)$, $SSP(c\neq 0)$ and $DSP(0,1)$ using consumer and producer quality levels. Jayalakshmi (2009) has presented a procedure for designing skip lot sampling plan of type SkSP-2 with STDS as reference plan involving minimum angle method between the lines formed by the points $(AQL, 1-\alpha)$, (AQL, β) and $(AQL, 1-\alpha)$, (LQL, β) are given. Kalaichelvi (2012) has studied the selection of skip-lot sampling plans for given $p1$, $p2$, α , and β involving producer and consumer risks with various reference plans. Suresh and Kavithamani (2013) have proposed the minimum angle approach between two points on the OC curve using the attribute sampling plan of SkSP-V with MRGS plan as reference plan. Recently, a new type of skip-lot sampling plan called SkSP-R was developed by Balamurali.et.al (2014) based on the principle of continuous sampling procedure and resampling scheme for the quality inspection of continuous flow of bulk products. The design parameters are determined so as to minimize the average sample number while the specified producer risk and the consumer risks are satisfied. The concept of multiple

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dependent (or deferred) state sampling (MDS) was introduced by Wortham and Baker (1976). The MDS sampling plan belongs to the group of conditional sampling procedures. In these procedures, acceptance or rejection of a lot is based not only on the sample from that lot, but also on sample results from past lots or from future lots (in the case of deferred state sampling). The operating procedure and characteristics of the attributes MDS sampling plan can be found in Wortham and Baker (1976) and this plan was studied further by Vaerst (1982). Subramani.K and Govindaraju.K (1990) have presented tables for the selection of multiple deferred state MDS – 1 sampling plan for given acceptable and limiting quality levels using Poisson distribution.

Skip lot sampling plan SkSP-R

The SKSP-R plan is a new system of skip lot sampling procedure based on the principle of continuous sampling procedure and resampling scheme for the quality inspection of continuous flow of bulk products. The SkSP –R plan uses the reference plan which is similar to the SkSP-2 plan of Perry (1973). The operating procedure of SkSP – R plan is as follows.

Operating procedure for SkSP-R plan

The operating procedure of SkSP-R are as follows

- Begin the procedure with normal inspection by applying specified reference plan (for instance MDS (0,1)). During normal inspection, lots are inspected one by one in order of being submitted.
- When i consecutive lots are accepted based on the reference plan under normal inspection, discontinue the normal inspection and switch to skipping inspection.
- During the skipping inspection, inspect only a fraction f of lots selected at random by applying reference plan. The skipping inspection is continued until a sampled lot is rejected.
- When a lot is rejected, after s consecutively sampled lot have been accepted, then go for re-inspection procedure for the immediate next lot as in step (5) given below.
- During re-inspection procedure, perform the inspection using the reference plan. If the lot is accepted, then continue the skipping inspection. On non-acceptance of the lot, re-inspection is done m times and the lot is rejected if it has been accepted on (m-1)st resubmission.
- If a lot is rejected on the re-inspection scheme, then we immediately revert to normal inspection.
- Replace or correct all non-conforming units found with conforming units in the rejected lots.

Operating procedure for MDS(r, b) plan

- For each lot select a sample of n units and test each unit for conformance to specified requirements
- Accept the lot if d (the observed number of observation of defectives) is less than or equal to r, reject the lot if d greater than or equal to r+b.
- If $r+1 \leq d \leq r+b$, accept the lot if the forthcoming m lots in succession are all accepted.

Operating procedure for MDS (0,1) plan

A multiple deferred State sampling plan of Wortham and Baker (1976) with r=0 and b=1 is operated as follows:

- From each lot, take a random sample of units and observe the non-conforming units d
- If d=0, accept the lot; if d>1, reject the lot. If d=1, accept the lot, provided the forthcoming m₂ lots in succession are all accepted (previous m₂ lots in case of multiple dependent state sampling).

The probability of acceptance based on Poisson model is,

$$P_a(p) = e^{-np} + npe^{-np} e^{-npm_2}$$

The operating characteristic function of MDS(r, b) is as follows

$$P = P_a(p) = P_{a,r}(p) + [P_{a,r+b}(p) - P_{a,r}(p)][P_a(p)]^{m_2}$$

When the parameter r=0, b=1. The probability of acceptance will be

$$P_a(p) = e^{-np} + npe^{-np} e^{-npm_2}$$

Designing method using minimum angle criteria

Norman Bush *et al.* have considered two points on the OC curve as (AQL, 1-α) and (LQL, 0.05) for minimizing the consumer’s risk. Here another approach of minimization of angle between the lines joining the points (AQL, β), (AQL, 1-α) and (AQL, 1-α), (LQL, β) was given by singaravelu (1993). Applying this method one can get a better plan which has an OC curve approaching to the ideal OC curve.

The formula for tanθ is given as

$$\tan \theta = \frac{\text{Oppositeside}}{\text{Adjacentside}}$$

$$n \tan \theta = \frac{np_2 - np_1}{P_a(p_1) - P_a(p_2)}$$

Where p₁= AQL and p₂=LQL. This may also be expressed as

$$n \tan \theta = \frac{np_2 - np_1}{1 - \alpha - \beta}$$

Using this formula, the angle θ is minimized for the given np₁ and np₂ values.

Selection procedure for sksp-r with multiple deferred state sampling plan mds (0,1)

Table 1 can be used for obtaining plan parameters with the minimum tangent angle (ntanθ) between the lines formed by the points (AQL, 1-α), (AQL, β) and (AQL, 1-α), (LQL, β).

One can find the sampling plan from the tables with minimum tangent angle (ntanθ) by the following procedures:

- Compute the operating ratio p_2/p_1
- With the computed values of p_2/p_1 enter the value from the table headed by p_2/p_1 this is equal to or just greater than the computed ratio.
- The sample size is then obtained as $n = np_1/p_1$, since θ is known, the parameter n can be computed.
- Thus the minimum angle can be found as $\theta = \tan^{-1}\{(n \tan \theta)/n\}$

Selection of plan for given i, f, k, p₁, and p₂

To select a plan for given i,f,k,m,p₁ and p₂, first calculate the operating ratio p_2/p_1 . From Table 1, locate the nearest OR value and the associated np₁ values. The other associated ntanθ, α %, β% can be recorded from the Table 1. Now,

$$\theta = \tan^{-1}(ntan\theta)/n.$$

For example,

For given p₁ = 0.01, p₂ = 0.07

- The Operating Ratio $p_2/p_1 = 7$
- Corresponding to obtain operating ratio, the parametric values are , f = 2/7 i=1, k=2, m₁ = 2, m₂=1
- $n=np_1/p_1 = 0.4292/0.01 = 42.92 \approx 43$

The associated sets of values corresponding to the computed OR of 7 are

1. ntanθ = 3.03141 α = 0.05 β = 0.10 θ = 1.66845
2. ntanθ = 2.74117 α = 0.05 β = 0.01 θ = 1.626922
3. ntanθ = 2.89517 α = 0.01 β = 0.10 θ = 1.649882
4. ntanθ = 2.62929 α = 0.01 β = 0.01 θ = 1.608759

The optimum plan is (43, 2/7, 1, 2, 2, 1) with the minimum angle $\theta = 1.608759$.

For $\alpha = 0.01, \beta = 0.01$

Construction of Tables

The probability of acceptance for SkSP-R with MDS(0,1) as reference plan designated as SkSPMDS-R plan (0,1) will be $P_a(f,i,k,r,b,m_1,m_2) = P_a(p)$

$$= \frac{fP + (1-f)P^i + fp^k(P^i - P)(1-Q^{m_1})}{f(1-p^i)[1 - P^s(1-Q^{m_1}) + P^i(1 + fPQ^s)]} \quad (1)$$

Where $P = e^{-np} + np_1e^{-np}e^{-npm_2}$ (2)

Where P is the Operating Characteristic function for MDS(r,b)

$$P = P_a(p) = P_{a,r}(p) + [P_{a,r+b}(p) - P_{a,r}(p)] [p_a(p)]^m$$

When the parameter r = 0 and b = 1
The probability of acceptance will be

$$P_a(p) = e^{-np} + np_1e^{-np}e^{-npm_2} \quad (3)$$

Where p is the operating ratio of p₂/p₁ and np₁ are known then np₂ can be obtained as

$$np_2 = (p_2/p_1) (np_1).$$

Table 1. Parametric values of SkSPMDS-R plan (0,1) through minimum angle method

F	i	K	m1	m2	np1	np2	ntanθ	ntanθ	ntanθ	ntanθ	OR=P2/P1
							α=0.05,β=0.10	α=0.05,β=0.01	α=0.01,β=0.10	α=0.01,β=0.01	
1/3	1	2	2	1	0.4	2.9	2.91800	2.63862	2.78685	2.53092	7.203852
1/5					0.5	3.3	3.27753	2.96372	3.13022	2.84276	6.351325
1/7					0.6	3.6	3.50859	3.17266	3.35090	3.04316	5.824947
2/5					0.4	2.7	2.79776	2.52989	2.67202	2.42663	7.668817
2/7					0.4	3	3.03141	2.74117	2.89517	2.62929	7.003495
3/5	1	3	2	1	0.3	2.4	2.52106	2.27968	2.40775	2.18663	8.691673
	2				0.3	2.1	2.12729	1.92362	2.03169	1.84510	7.490309
	3				0.3	2	2.06082	1.86351	1.96820	1.78745	7.287509
	4				0.3	2	2.05988	1.86266	1.96730	1.78663	7.284637
2/7	5				0.3	2	2.06459	1.86691	1.97180	1.79071	7.395408
	3	1	2	1	0.4	2.1	1.97082	1.78213	1.88225	1.70939	5.013416
	2				0.4	2.1	1.94635	1.76000	1.85888	1.68816	5.074877
	3				0.4	2.1	1.95106	1.76426	1.86337	1.69224	5.178382
	4				0.4	2.1	1.95776	1.77032	1.86978	1.69806	5.263643
1/5	5				0.4	2.1	1.96341	1.77543	1.87517	1.70296	5.33143
	2	1	1		0.4	2	1.84306	1.66660	1.76022	1.59857	4.961062
	2				0.4	2	1.86588	1.68723	1.78202	1.61837	4.624314
	3				0.4	2.1	1.90435	1.72202	1.81876	1.65173	4.605926
	4				0.5	2.1	1.95271	1.76574	1.86494	1.69367	4.677005
2/5	5	3	2	1	0.5	2.1	1.99624	1.80511	1.90652	1.73143	4.754813
	2				0.3	2	1.98506	1.79500	1.89584	1.72173	6.119235
	2				0.3	1.8	1.83941	1.66330	1.75674	1.59541	6.990421
	3				0.2	1.8	1.83706	1.66117	1.75449	1.59337	7.974096
	4				0.2	1.8	1.85224	1.67489	1.76899	1.60653	8.74422
				5	0.2	1.8	1.86706	1.68830	1.78315	1.61939	9.396825

Acceptance sampling plan have been widely used in industry to determine whether the manufactured item satisfy the pre-specified quality levels or not. At this point, an enterprise must have to take a decision for accepting or rejecting the lots in accordance with randomly chosen units. The work presented in this paper are mainly related to the new procedure for selection of Skip-lot sampling plan of type SkSP-R with MDS (0,1) plan through minimum angle criteria involving producer's and consumer's quality levels

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